Some Circuit Properties and Applications of *n-p-n* Transistors

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Shockley, Sparks, and Teal have recently described the physical properties of a new kind of transistor. Preliminary studies of circuit performance show that it is a stable, high gain, quiet amplifier of considerable practical interest. This paper analyzes the performance of a few early experimental units.

Introduction

Almost two years ago, W. Shockley[1, 2] first published tbe theory of a transistor made from a single piece of germanium in which the conductivity type varies in such a way as to produce two rectifying junctions. Since that time, M. Sparks, G. K. Teal and others at the Bell Telephone Laboratories[3,4] bave contributed notably to the physical realization of this device.

Recently Sparks has produced a number of *n-p-n* transistors and has found their behavior to be closely in accord with Sbockley's theory.[4] Preliminary circuit studies on tbese devices bave shown tbat in several respects tbeir performance is remarkable. In view of this, our transistor development group has undertaken to produce small quantities of *n-p-n* transistors in a form suitable for incorporation in working circuits.

This paper will deal principally witb tbe circuit aspects of the *n-p-n* transistor by presenting and analyzing performance data on a small number of experimental units. For a discussion of the solid state physics of its design and operation tbe reader is referred to the previously mentioned works of Sbockley, Sparks, and Teal.

Outstanding Properties

Before getting lost in a maze of detail, it seems wortbwhile to list and mention briefly the salient features of this new transistor. They are:

1. *Relatively low noise figure.* Most of the units measured so far have a noise figure between 10 and 20 db at 1000 cps.

2. *Complete freedom from short-circuit instability.* The input and output impedances are always positive whether the transistor is connected grounded-emitter, grounded-base, or grounded collector. This permits a great deal of freedom in circuit design and makes it possible, by choosing the appropriate connection, to obtain a considerable variety of input and output impedances.

3. *High gain.* Power gains of the order of 40 to 50 db per stage have been obtained

4. *Power handling capacity and efficiency.* The design can readily be varied to permit the required amount of power dissipation up to at least two watts. Furthermore the static characteristics are so nearly ideal that Class A efficiencies of 48 or 49 out of a possible 50% can be realized. The efficiencies for Class B and Class C operation are correspondingly high.

5. *Ruggedness and small size.* The germanium part of the transistor is enclosed in a hard plastic bead about $\frac{3}{16}$ inch in diameter. Inside the bead three connections are mechanically as well as electrically fastened to the germanium and are brought out as pigtails through the bead. This gives a very sturdy unit.

6. *Freedom from microphonics.* Vibration tests in the audio frequency range indicate that these devices are relatively free from microphonic noise.

7. *Limited frequency response.* Collector capacitance limits the frequency response at full gain to a few kilocycles. By using a suitable impedance

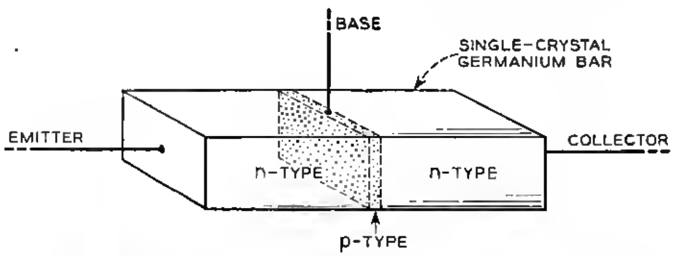

Fig. 1—The heart of an *n-p-n* transistor is a tiny bar of germanium to which three mechanically strong electrical connections are made.

mismatch it is possible to maintain the frequency response flat to at least one megacycle while still obtaining a useful amount of gain.

8. *Operation with exceedingly small power consumption.* Perhaps the most remarkable feature of these transistors is their ability to operate with exceedingly small power consumption. The best example of this to date is an audio oscillator which requires for a power supply only 6 microamperes at 0.1 volts. This represents 0.6 microwatts of power which contrasts sharply with the million or more microwatts required to heat the cathode of an ordinary receiving-type vacuum tube.

Physical Appearance and Construction

Figure 1 shows schematically the configuration of an *n-p-n* transistor. The small bar of single crystal germanium contains a thin layer of *p*-type interposed between regions of *n*-type. Mechanically strong ohmic connections are made to the three regions as indicated and brought out through a

hard plastic bead. A finished transistor is shown in the photograph of Fig. 2. It should be pointed out that Fig. 1 is not drawn to scale and that the p-layer may be less than a thousandth of an inch thick.

Static Characteristics

A great deal of information about the low frequency performance of a transistor can be obtained from a set of static characteristics such as those shown in Fig. 4. Curves of this sort are obtained simply by connecting suit-

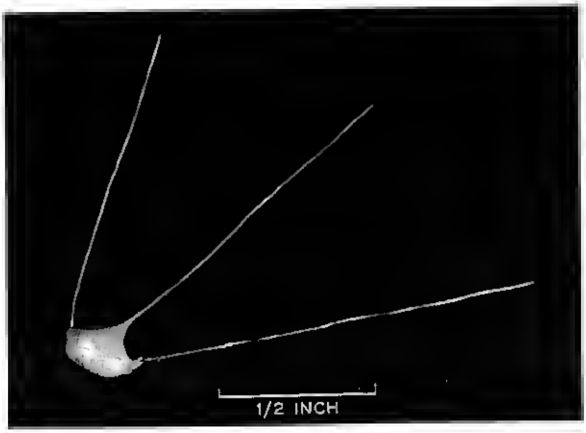

Fig. 2—A beaded n-p-n transistor.

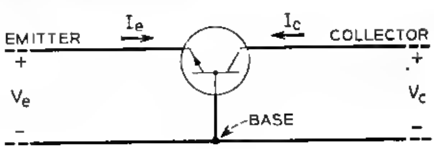

Fig. 3—The symbol for a p-type transistor on which the convention of signs for currents and voltages is indicated.

able current sources to the emitter and collector circuits of the transistor and measuring the resulting voltages. The currents are called positive when they flow into the emitter and collector as shown and the voltages are called positive when they have the signs shown in Fig. 3.

Let us first examine these curves with an eye to finding out what kind of voltage and current supplies are needed to bias the transistor into the range in which it can amplify. To make this easy, that part of the characteristics which lies within the normal operating range has been shown as solid lines and that part of the characteristics corresponding to cutoff has been shown as dotted lines.

Note from the upper set of curves that V_c is positive in the operating range. This means that the collector must be biased positive with respect to the

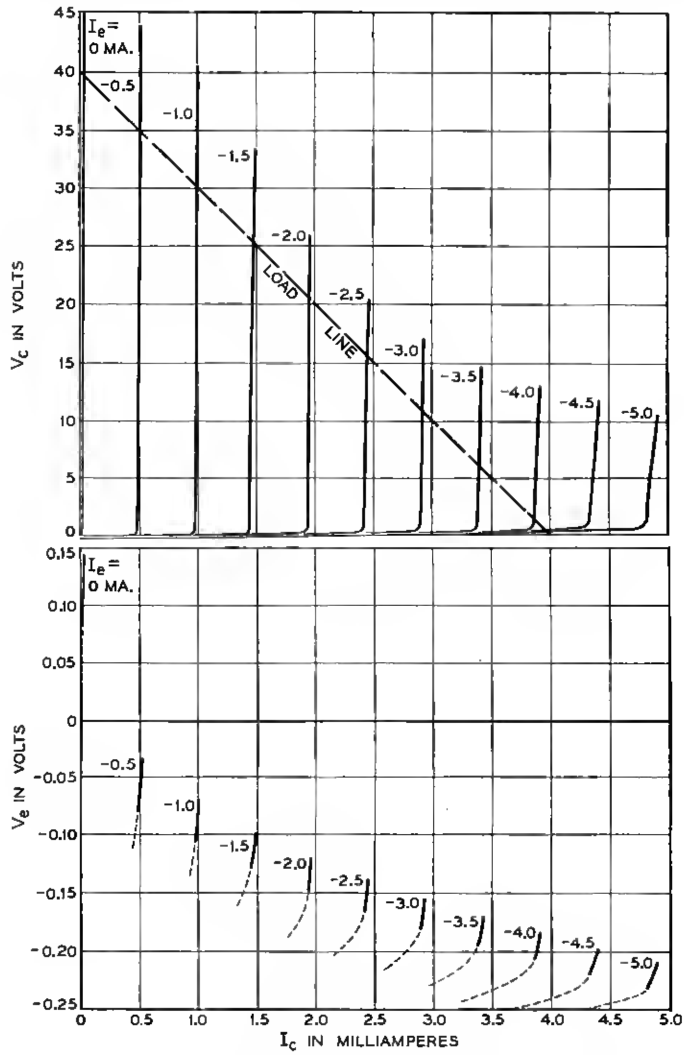

Fig. 4—Static characteristics of an *n-p-n* transistor.

base. For this particular transistor a bias voltage anywhere between about 0.1 volts and 35 volts is suitable. Note also that all the curves on this plot correspond to negative emitter currents. This means that the emitter must

be biased in such a way that current flows out of the emitter into a suitable current supply. Furthermore, the collector current corresponding to any given emitter current can be seen to be almost equal in magnitude to the emitter current. Since these two currents are opposite in sign, this means that most of the current which flows into tbe collector leaves by way of the emitter witb tbe result tbat the current in the base circuit is very small.

Suppose that the collector is held at a constant positive voltage as, for example, by connecting a battery hetween collector and base (witb a transformer winding in series, perhaps). Now if a negative current is forced into the emitter hy a battery and resistance connected in series between emitter and base, the collector current can he controlled hy varying the emitter current and will always be approximately equal in magnitude to the emitter current. Suitable collector currents for this particular transistor range from about 20 microamperes to about five milliamperes.

The exact choice of collector current and voltage within the ranges mentioned ahove will be dictated largely by the amount of power output required. The more power output required, the more current and voltage will he needed from the power supply. Since the collector circuit efficiency cannot exceed the theoretical limit of 50% in Class A operation, the signal power output cannot exceed half the power supplied by the battery. This means, for example, that if the collector is worked at 20 volts and 2 milliamperes the Class A power output cannot exceed 20 milliwatts.

From tbe lower plot of Fig. 4 it is possible to obtain information about tbe hias voltage required for the emitter. Note, first, that tbe entire emitter voltage plot corresponds to a very small range of emitter voltages near zero and, furthermore, that the part of the characteristics corresponding to the operating range covers only a few thousandths of a volt. This means that if the collector voltage is held constant very small changes in emitter voltage will produce fairly large changes in collector current, or if the collector current is held constant very small changes in emitter voltage will produce relatively enormous changes in collector voltage. This at once suggests the use of this transistor as a d-c. amplifier between a low impedance source and a higb impedance load. In this application, voltage stepup of the order of 10,000 times is possible.

The very great sensitivity of the collector circuit to emitter voltage suggests, however, tbat for a-c. amplifiers one should use a current source as an emitter bias supply. This can be obtained from a battery and a large resistance in series. Furthermore, since the emitter voltage is always nearly zero, the emitter current can be calculated in advance by dividing the battery voltage by the value of the series resistance (provided, of course, that

the supply voltage is large compared to the few hundredths of a volt drop across the emitter circuit).

One can also draw some interesting conclusions from the static characteristics about the large signal operation of the transistor. If the load is resistive, the instantaneous operating point will swing up and down along a straight line such as the load line shown in the upper plot of Fig. 4. This particular load line corresponds to an a-c. load resistance of 10,000 ohms. Suppose that the steady collector biases are 20 volts and 2 milliamperes so that the drain from the power supply is 40 milliwatts. Now consider the permissible swings of collector voltage and current. Since the collector characteristics are quite straight and evenly spaced over a wide range of current and voltage values, the output signal can swing nearly down to zero collector volts and nearly up to zero collector current without distortion. The limit on the lower end is imposed by the fact that the collector characteristics begin to be curved when V_c is less than about 0.1 volts; and the limit on the upper end is imposed by the fact that the collector current does not drop completely to zero when I_e drops to zero. The lower limit of collector current is, in this case, about 50 microamperes and, since this amount of current in 10,000 ohms corresponds to 0.5 volts, this means that the instantaneous collector voltage is limited to swings between 39.5 volts and 0.1 volts. Starting from a quiescent value of 20 volts, the permissihle positive swing is then 19.5 volts and the permissible negative swing is 19.9 volts. Reducing the quiescent voltage to 19.8 volts (and keeping the same load line) makes it possihle to ohtain a peak swing of 19.7 volts which corresponds to 19.45 milliwatts of signal delivered to the load. This gives a collector circuit efficiency of 48.5% out of a possihle 50%. Some transistors take even less collector current when the emitter current is zero and hence permit even higher efficiencies.

These computations of efficiency have all heen based on the assumption of sinusoidal *current* applied to the emitter. It will be shown in a later section that the emitter resistance varies with emitter current, however, and this means that to realize high efficiency with low distortion it is necessary to drive the emitter from a high impedance source.

Operation with Small Power Consumption

For small signal applications the transistor represented by the characteristics of Fig. 4 can deliver useful gain at very much lower voltages and currents than those used in the example above. In order to show this, the characteristics of Fig. 5 have been plotted for a range of collector voltage extending up to only 2 volts and for a range of collector currents extending

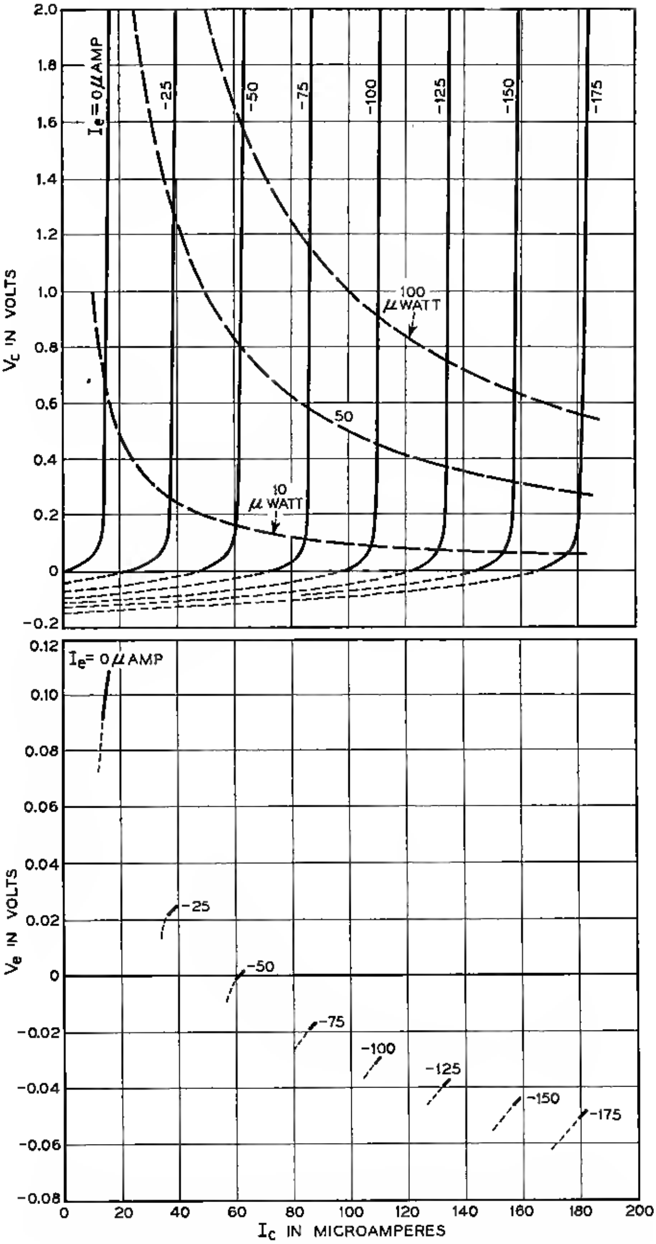

Fig. 5—Static characteristics showing behavior at very low applied voltages and currents.

up to only 200 microamperes. It can be seen from the upper plot that the collector circuit characteristics are still quite usefully straight and evenly spaced in this micro-power range. In fact, for small signal operation it is sufficient to use a collector voltage only a little in excess of 0.1 volts and a collector current a little in excess of 20 microamperes. This means that the power required to bias the collector into the operating range amounts to only a few microwatts. Contours are shown for 10, 50, and 100 microwatts of power supply.

This ability of the transistor to work with extremely small power consumption is one of its most striking and perhaps most important features

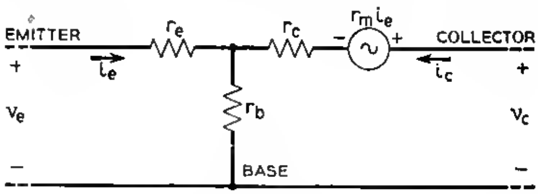

Fig. 6—The low-frequency equivalent circuit of a transistor.

When one considers that the total power consumption of a single transistor stage can be smaller by many thousands of times than the power required to heat the cathode in a vacuum tube, it is obvious that the advent of this device will make possible many new kinds of application.

VARIATION OF TRANSISTOR PROPERTIES WITH OPERATING POINT

Ryder and Kircher[5] have shown that it is convenient to analyze the small signal properties of a transistor at low frequencies in terms of the equivalent circuit of Fig. 6 where r_e is called the emitter resistance, r_b is called the base resistance, and r_c is called the collector resistance. The internal generator, $r_m i_e$, is the active part of the circuit and in this respect corresponds to the familiar μe_g of vacuum tube circuit theory. It is the purpose of this section to show what values these quantities have for a particular *n-p-n* transistor and to show how they vary with the applied biases. This will form a basis for the next section in which these quantities will be used to compute such things as the input and output impedances and the gains of various transistor connections.

Ryder and Kircher have shown that these four r's can be obtained directly from static characteristics such as those shown in Fig. 4 and Fig. 5. In the case of *n-p-n* transistors, however, the magnitudes of these quantities are such that it is difficult to obtain satisfactory accuracy in this way and it has been more convenient to measure the 4-pole r's by a-c. methods.

These measurements have shown that all of the r's are, to a first approximation, independent of collector voltage so long as the collector voltage is above a few tenths of a volt and so long as the total dissipation is small enough to prevent appreciahle heating of the transistor.

In view of this fact it is perhaps sufficient to show how these quantities vary with emitter current for a moderate fixed value of collector voltage. Figures 7 and 8 show that r_c and r_m are very nearly equal and that they tend to decrease as I_e increases. Theoretically r_m and r_c should hoth he infinite. The fact that they reach values as low as 10 megohms in this case is a measure of the imperfection in technique of fabricating the transistor. Values as high as 60 megohms have heen achieved in the laboratory.

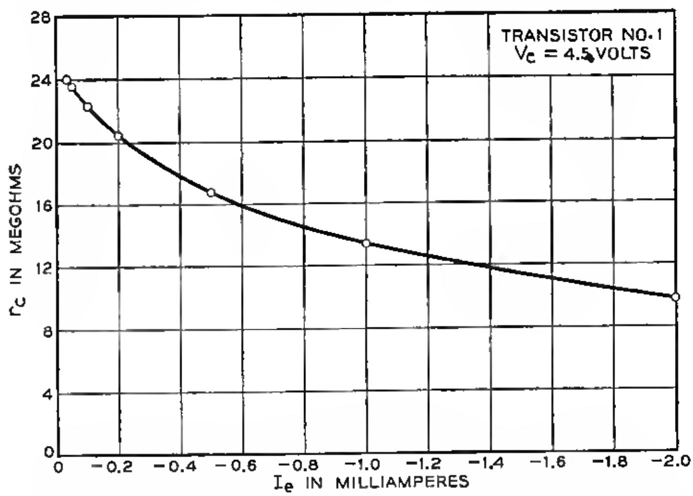

Fig. 7—The variation of collector resistance with emitter current at a fixed value of collector voltage.

Figure 9 shows that r_b in this transistor is approximately 240 ohms and is independent of I_e.

Figure 10 shows that r_e decreases with increasing emitter current, ranging from ahout 500 ohms at 50 microamperes down to about 5 ohms at 5 milliamperes. Shockley[4] has shown that r_e should be given by

$$r_e = \frac{kT}{qI_e} \tag{1}$$

where q is the charge on an electron, k is Boltzman's constant, T is the Kelvin temperature and I_e is the emitter current. When the temperature

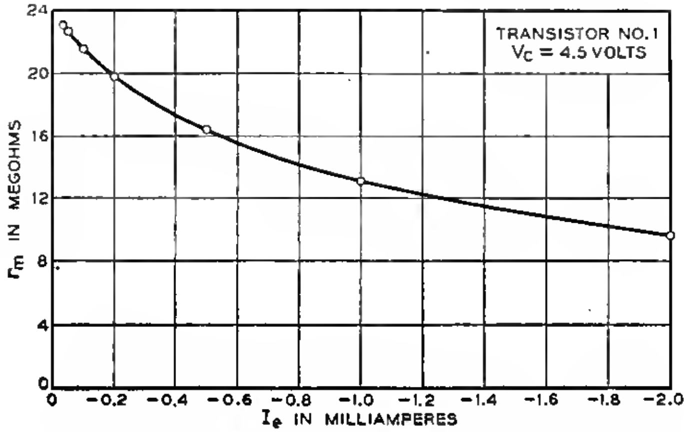

Fig. 8—Variation of r_m with emitter current at fixed collector voltage.

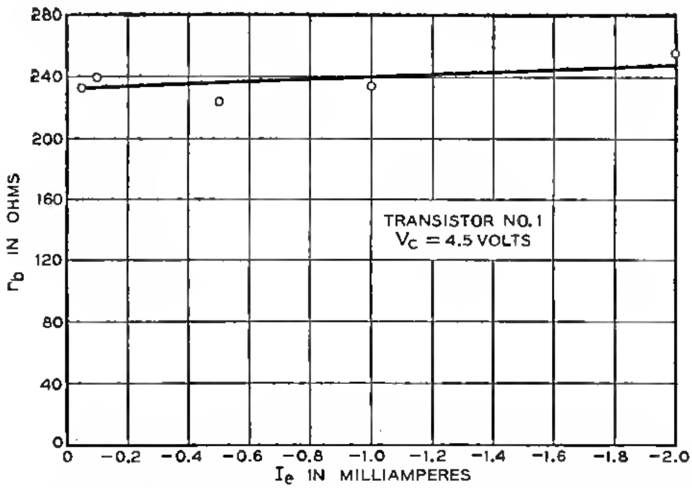

Fig. 9—Variation of base resistance with emitter current. Scatter of the data indicates that the measurements were not accurate.

is about 80° F., this reduces to

$$r_e = \frac{25.9}{I_e} \tag{2}$$

where I_e is measured in milliamperes. Within experiment error, values of r_e computed from this relation agree perfectly with the measured curve shown in Fig. 10.

Figure 11 introduces a new quantity, α, the current amplification factor of the transistor. This quantity is defined by the equation

$$\alpha = \frac{r_m + r_b}{r_c + r_b} \tag{3}$$

Since r_m and r_c are both very large compared to r_b, α is approximately equal to the ratio of r_m to r_c. It will be shown in a later section that this quantity is important in determining some of the circuit properties of the transistor and that many of the circuit properties become more desirable as α approaches unity.

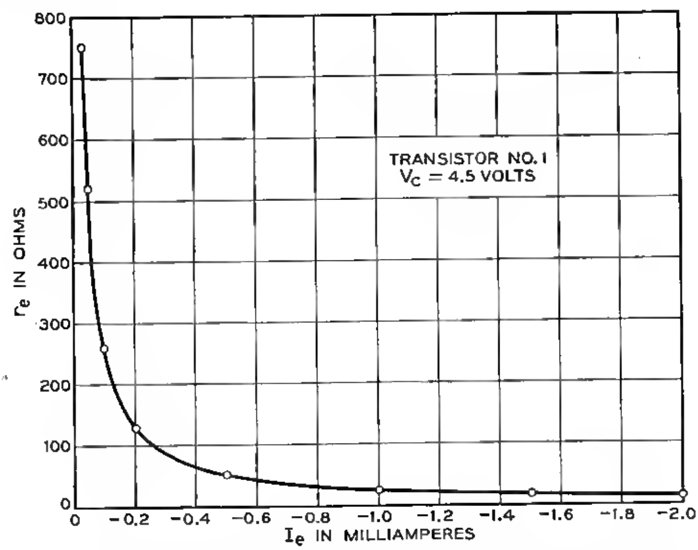

Fig. 10—The emitter resistance is inversely proportional to emitter current.

It can be seen from Fig. 11 that in this transistor α is approximately equal to 0.98 and that it increases slightly with increasing emitter current. The highest value of α so far achieved is 0.9965.

Those units which have been made in the laboratory so far show considerable variation in some of the properties, but this is partly due to the fact that changes have been made deliberately to test one aspect or another of Shockley's theory. The data in table I are presented to indicate what properties have been achieved to date. The collector capacitance C_c will be discussed in a later section.

General Considerations and Formulae

It is a consequence of the fact that α is always less than unity in this structure that these transistors are unconditionally stable with all termina-

tions. This means that stability considerations do not prevent working with matched terminations. Furthermore it is possible to obtain a variety of input and output impedances by connecting the transistor as a grounded-emitter, grounded-base, or grounded-collector stage. It is the purpose of this section to give some idea of the characteristics of these various stages and to show in each case at least one way of supplying the required biases and couplings to the stage.

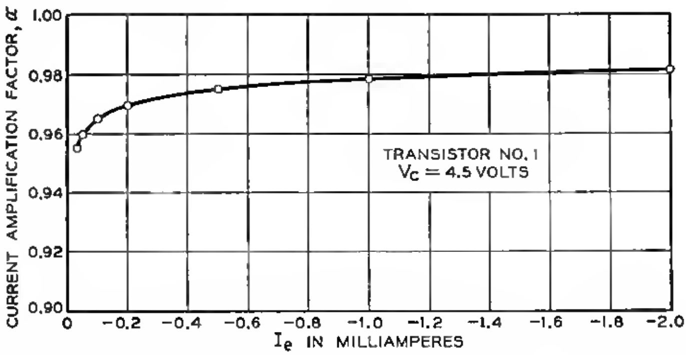

Fig. 11—The current amplification factor, α, increases slightly with increasing emitter current. Note the expanded scale for α.

TABLE I

CONSTANTS FOR VARIOUS TRANSISTORS MEASURED AT $V_c = 4.5$ v., $I_e = 1.0$ ma.

Transistor No.		I	II	III	IV	V
r_e	(ohms)	25.9	31.6	33.1	30.2	38.8
r_b	(ohms)	240	44	300	3070	180
r_c	(megohms)	13.4	0.626	1.11	1.21	2.00
$r_c - r_m$	(megohms)	0.288	0.00387	0.0168	0.00422	0.0439
α		0.9785	0.9936	0.9848	0.9965	0.9780
C_c	($\mu\mu$f.)	7	7.7	18.9	27.9	21.2

It will be convenient to begin by writing down general relationships which will apply to all the possible connections. To this end let the transistor be represented by the box in Fig. 12. At low frequencies, the signal currents and voltages are related through the equations:

$$\begin{aligned} R_{11}i_1 + R_{12}i_2 &= v_1 \\ R_{21}i_1 + R_{22}i_2 &= v_2 \end{aligned} \tag{4}$$

If a generator of open circuit voltage v_g and internal resistance R_g is connected to the input terminals of the device as shown in Fig. 13, then

$$v_1 = v_g - i_1R_g \tag{5}$$

and if a load of resistance R_L is connected to the output terminals

$$v_2 = -R_L i_2 . \tag{6}$$

The equations for the circuit of Fig. 13 are, therefore,

$$\begin{aligned} (R_{11} + R_g)i_1 + R_{12}i_2 &= v_g \\ R_{21}i_1 + (R_{22} + R_L)i_2 &= 0 \end{aligned} \tag{7}$$

Solving for the voltage developed across the load ($= -R_L i_2$) gives

$$v_2 = \frac{R_L R_{21}}{(R_{11} + R_g)(R_{22} + R_L) - R_{12}R_{21}} v_g \tag{8}$$

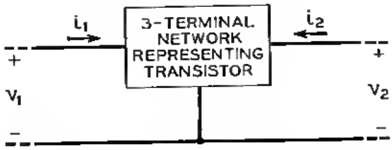

Fig. 12—A three-terminal network representing either grounded emitter, grounded base, or grounded collector connection of a transistor. Note the convention of signs.

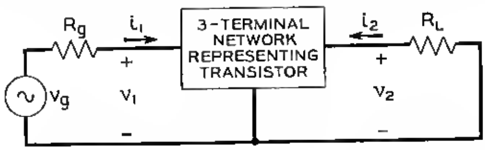

Fig. 13—The three-terminal network of Fig. 12 connected between a generator and a load.

The power gain in the circuit is the power delivered to the load (v_2^2/R_L) divided by the power available from the generator ($v_g^2/4R_g$). From equation (8), this gives

$$G = \frac{4R_g R_L R_{21}^2}{[(R_{11} + R_g)(R_{22} + R_L) - R_{12}R_{21}]^2} \tag{9}$$

The gain depends on R_g and R_L and will be maximum when these are chosen to match the input and output impedances of the transistor stage. But the input impedance depends on R_L and the output impedance depends on R_g in the following way:

$$\text{Input impedance} = R_i = R_{11} - \frac{R_{12}R_{21}}{R_{22} + R_L} \text{ and} \tag{10}$$

$$\text{Output impedance} = R_o = R_{22} - \frac{R_{12}R_{21}}{R_{11} + R_g} \tag{11}$$

If $R_i = R_g$ and $R_o = R_L$ then impedances are matched at the input and output terminals and the gain is a maximum. The conditions are:

Matched input impedance =

$$R_{im} = R_{11} \sqrt{1 - R_{12} R_{21}/R_{11} R_{22}}\,, \tag{12}$$

Matched output impedance =

$$R_{om} = R_{22} \sqrt{1 - R_{12} R_{21}/R_{11} R_{22}}\,, \tag{13}$$

Maximum available gain =

$$\text{M.A.G.} = \frac{R_{21}^2}{R_{11} R_{22}} \frac{1}{[1 + \sqrt{1 - R_{12} R_{21}/R_{11} R_{22}}]^2} \tag{14}$$

The Grounded Base Stage

In this and the following two sections we will put into equations (7) through (14) the appropriate 4-pole r's to obtain expressions for impedances and gains. As a numerical example we will substitute into the resulting equations the measured values of these r's for Transistor No. I working at V_c = 4.5v. and I_c = 1 ma. It must be understood that the numerical values may vary appreciably from transistor to transistor and that these numerical calculations are intended only for illustration and not as a basis for final circuit design. The numerical values to be used are

$$\begin{aligned} r_e &= 25.9 \text{ ohms} \\ r_b &= 240 \text{ ohms} \\ r_c &= 13.4\ (10)^6 \text{ ohms} \\ r_c - r_m &= 0.288\ (10)^6 \text{ ohms} \\ \alpha &= 0.9785 \end{aligned} \tag{15}$$

In this section it will be shown that the grounded base connection is suitable for working between a low impedance source and a high impedance load. The input impedance may be of the order of a hundred ohms and the output impedance of the order of one or more megohms. In this connection the current amplification is always less than unity but the voltage amplification may be very large indeed. Power gains of the order of 40 to 50 db can be obtained between matched impedances, and appreciable gains can still be obtained if the load resistance is reduced to a few thousand ohms (because the current gain is then almost equal to unity). In this case the gain of the stage is almost completely independent of those transistor properties

which tend to vary from unit to unit. This sort of stage does not produce a phase reversal.

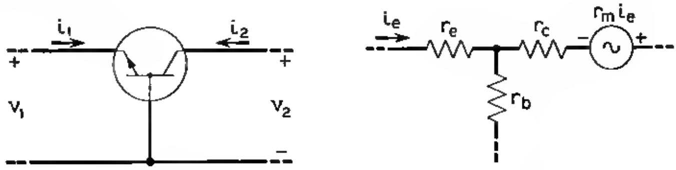

Fig. 14—The grounded base connection of a transistor.

For the grounded base stage shown in Fig. 14,

$$\begin{aligned} R_{11} &= r_e + r_b = 266 \text{ ohms} \\ R_{12} &= r_b = 240 \text{ ohms} \\ R_{21} &= r_m + r_b = 13.1\ (10)^6 \text{ ohms} \\ R_{22} &= r_c + r_b = 13.4\ (10)^6 \text{ ohms} \end{aligned} \tag{16}$$

$$\begin{aligned} \alpha &= \frac{r_m + r_b}{r_c + r_b} \\ &\doteq \frac{r_m}{r_c} \\ &= 0.9785 \end{aligned}$$

In this case if r_b is neglected by comparison with r_m and r_c, equation (8) leads to

$$v_2 = \frac{\alpha R_L\, v_g}{(r_e + r_b + R_g)(1 + R_L/r_c) - \alpha r_b}. \tag{17}$$

Since for these transistors $\frac{r_m}{r_c}$ $(\doteq \alpha)$ is always less than unity, the output voltage is in phase with the input voltage. Furthermore, if R_L is very high, the output voltage is enormous by comparison with the input voltage. For example, if $R_g = 0$ and R_L is infinite

$$v_2 = v_g \frac{r_m}{r_e + r_b}. \tag{18}$$

and for the numerical example this is

$$v_2 = 4.93\ (10)^4\, v_g.$$

To achieve this step-up would require a load impedance very large compared to 13 megohms, but even with more modest values of load impedance the voltage step-up is large.

If R_L is small compared to r_c, the second of equations (7) leads to

$$\begin{aligned} i_2 &= -\frac{r_m}{r_c} i_1 \\ &\doteq -i_1 \end{aligned} \tag{19}$$

and the current delivered to the load is approximately equal to the current which the generator delivers to the transistor.

From equations (10) and (11), the input and output impedances are

$$R_i = r_e + r_b - \frac{r_b(r_m + r_b)}{r_c + R_L + r_b} \tag{20}$$

$$R_o = r_c + r_b - \frac{r_b(r_m + r_b)}{r_e + r_b + R_g} \tag{21}$$

As the load impedance varies from zero to infinity, the input impedance varies from

$$\begin{aligned} R_i &= r_e + r_b\left[1 - \frac{r_m + r_b}{r_c + r_b}\right] \quad \text{for } R_L = 0 \\ &\doteq r_e + r_b(1 - \alpha) \\ &= 31.1 \text{ ohm} \end{aligned} \tag{22}$$

to

$$R_i = r_e + r_b = 266 \text{ ohms} \quad \text{for } R_L = \infty. \tag{23}$$

When $R_g = 0$, the output impedance is

$$R_o = r_c - \frac{r_b}{r_e + r_b}(r_m - r_e) \tag{24}$$

$$\begin{aligned} &\doteq r_c - \frac{r_b}{r_e + r_b} r_m \\ &= 1.56\ (10)^6 \text{ ohms.} \end{aligned} \tag{25}$$

As R_g increases to infinity

$$R_o = r_c + r_b = 13.4\ (10)^6 \text{ ohms.} \tag{26}$$

From equations (12) and (13), the matched input and output impedances are approximately

$$\begin{aligned} R_{im} &= (r_e + r_b)\sqrt{1 - \alpha r_b/(r_e + r_b)} \\ &= 91 \text{ ohms} \end{aligned} \tag{27}$$

$$\begin{aligned} R_{om} &= (r_c + r_b)\sqrt{1 - \alpha r_b/(r_e + r_b)} \\ &= 4.58(10)^6 \text{ ohms.} \end{aligned} \tag{28}$$

With matched impedances, the maximum availahle gain is

$$\begin{aligned} \text{M.A.G.} &= \frac{\alpha(r_m + r_b)}{r_e + r_b}\left[1 + \sqrt{1 - \alpha r_b/(r_e + r_b)}\right]^{-2} \\ &= 2.7\ (10)^4 \text{ or } 44.3 \text{ dh.} \end{aligned} \tag{29}$$

The matched output impedance of this stage is inconveniently high but a useful amount of gain can he maintained if R_L is reduced to a more rea-

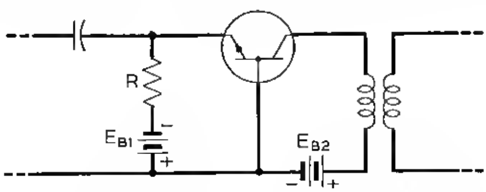

Fig. 15—One practical arrangement of a grounded base amplifier stage.

sonable value. For example, if $R_L = 200{,}000$ and $R_g = 25$, equation (9) gives

$$G = 5.3\ (10)^3 \text{ or } 37.2 \text{ db.}$$

If stages of this sort are to he cascaded, a step-down transformer must be used to couple each collector to the following emitter. Otherwise, since the current amplification factor of the transistor is slightly less than unity, the gain per stage will also he slightly less than unity.

One practical arrangement of a grounded hase stage would he as shown in Fig. 15. The required value of R will he approximately

$$R = \frac{E_{B1}}{I_c} \tag{30}$$

where I_c is the desired collector current and E_{B1} is the voltage of the emitter-hias hattery. For operating at $I_c = 1$ ma, for example, $E_{B1} = 1.5$ v and $R = 1500$ ohms would be suitable.

The Grounded Emitter Stage

For many applications the grounded emitter connection is more desirable than either of the other two. The power gains which can he ohtained are high—of the order of 50 db—and the interstage coupling problem is simplified by the fact that the input impedance is somewhat higher than that of the grounded base stage while the output impedance is very much lower. The input impedance may he of the order of a few hundred ohms and the output impedance of the order of a few hundred thousand ohms. Both voltage and current amplification are produced (with a phase reversal) and gains of the order of 30 db or more per stage can be obtained without the

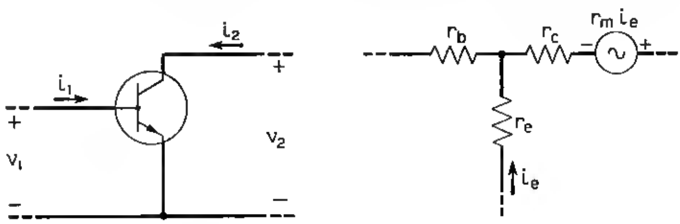

Fig. 16—The grounded emitter connection of a transistor and the equivalent circuit

use of interstage coupling transformers. The input and output impedance depend very critically on α and may vary appreciably from unit to unit

For this connection, which is indicated schematically in Fig. 16,

$$\begin{aligned} R_{11} &= r_e + r_b = 266 \text{ ohms} \\ R_{12} &= r_e = 25.9 \text{ ohms} \\ R_{21} &= r_e - r_m = -13.1\ (10)^6 \text{ ohms} \\ R_{22} &= r_e + r_c - r_m = 0.288\ (10)^6 \text{ ohms} \end{aligned} \tag{31}$$

Putting these values into equation (8) shows v_2 is always opposite in sign compared with v_g, that is, that the grounded emitter stage produces a phase reversal as does the grounded cathode vacuum tuhe.

If R_L is infinite and $R_g = 0$

$$\begin{aligned} v_2 &= v_g \frac{r_e - r_m}{r_e + r_b} \\ &= -4.93\ (10)^4\ v_g \end{aligned}$$

which is the same as for the grounded base stage. But if $R_L = 0$

$$i_2 = \frac{r_m - r_e}{r_e + r_c - r_m} i_1 \tag{32}$$

$$\doteq \frac{\alpha}{1 - \alpha} i_1$$
$$= 45.5\ i_1. \tag{33}$$

Thus it is seen that the grounded emitter amplifier can produce quite appreciable current amplification—particularly so when α approaches unity.

The input impedance to the stage is

$$R_i = r_e + r_b + \frac{r_e(r_m - r_e)}{r_e + r_c - r_m + R_L}. \tag{34}$$

When $R_L = 0$ this reduces to

$$R_i = r_b + r_e \frac{1}{\dfrac{r_e}{r_c} + 1 - \dfrac{r_m}{r_c}} \tag{35}$$

$$\doteq r_b + r_e \frac{1}{1 - \alpha}$$
$$= 1440 \text{ ohms}. \tag{36}$$

As R_L increases to infinity, the input impedance decreases to $r_e + r_b$ which, for the numerical example, is 266 ohms.

The output impedance is

$$R_o = r_e + r_c - r_m + \frac{r_e(r_m - r_e)}{r_e + r_b + R_g} \tag{37}$$

When $R_g = 0$, this gives

$$R_o = r_c - \frac{r_b}{r_e + r_b}(r_m - r_e) \tag{38}$$

$$\doteq r_c\left[1 - \frac{r_b}{r_e + r_b}\alpha\right]$$
$$= 1.56\ (10)^6 \text{ ohms} \tag{39}$$

As R_g increases to infinity, R_o decreases to

$$R_o = r_e + r_c - r_m$$
$$= 0.288\ (10)^6 \text{ ohms}. \tag{40}$$

The matched input and output impedances are

$$R_{im} = (r_e + r_b)\sqrt{1 + r_e(r_m - r_e)/(r_e + r_b)(r_e + r_c - r_m)} \tag{41}$$

$$= 619 \text{ ohms and}$$

$$R_{om} = (r_e + r_c - r_m)\sqrt{1 + r_e(r_m - r_e)/(r_e + r_b)(r_e + r_c - r_m)} \tag{42}$$

$$= 0.671\ (10)^6 \text{ ohms}$$

As α increases toward unity the matched input impedance increases and the matched output impedance decreases. They approach the limits

$$R_{im} = \sqrt{(r_b + r_c)(r_e + r_b)} \tag{43}$$

$$R_{om} = r_e\sqrt{(r_b + r_c)/(r_e + r_b)} \tag{44}$$

$$\text{as } \alpha \rightarrow 1.$$

If r_m in the transistor of our numerical examples could be increased to exactly the value of r_c ($\alpha = 1$) then the matched impedances would be

$$R_{im} = 59{,}700 \text{ ohms}$$

$$R_{om} = 5{,}800 \text{ ohms}$$

From this example, it is seen that tbe impedances vary rapidly with α as α approaches unity.

With matched impedances, the maximum available gain from the grounded emitter stage is

$$\text{M.A.G.} = \frac{(r_e - r_m)^2}{r_e r_c}\left[\sqrt{\left(1 + \frac{r_b}{r_e}\right)\left(\frac{r_e}{r_c} + 1 - \frac{r_m}{r_c}\right)} + \sqrt{1 + \frac{r_b}{r_e}\left(\frac{r_e}{r_c} + 1 - \frac{r_m}{r_c}\right)}\right]^{-2} \tag{45}$$

$$= 2.02\ (10)^5 \text{ or } 53 \text{ db}$$

When α is exactly unity this expression reduces to r_c/r_e provided r_e and r_b are small compared with r_c. For values of α which are enough smaller than unity so that

$$\frac{r_e}{r_c} \ll 1 - \alpha$$

the expression for maximum available gain reduces to the approximate expression

$$\text{M.A.G.} = \alpha(r_m/r_e)\left[\sqrt{(1 - \alpha)\frac{r_b}{r_e}} + \sqrt{1 + (1 - \alpha)\frac{r_b}{r_e}}\right]^{-2} \tag{46}$$

base and ground as shown, for example, in Fig. 18. Since the base floats at a positive potential with respect to ground, tbis circuit produces a base current of the right sign to decrease the collector current. As the value of the series resistance is decreased to zero, the collector current decreases to

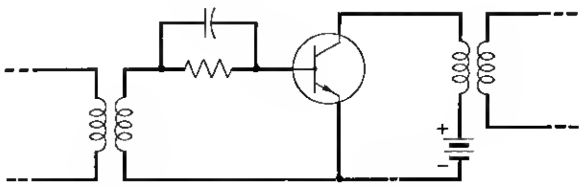

Fig. 18—Modification of Fig. 17 to obtain lower collector current.

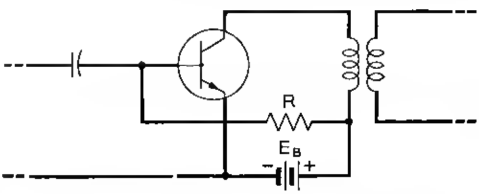

Fig. 19—Modification of Fig. 17 to obtain higher collector current.

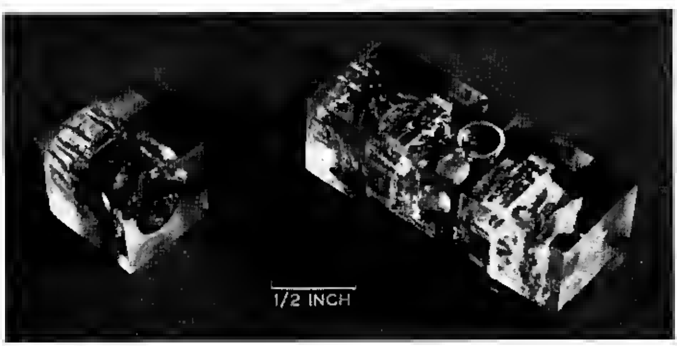

Fig. 20—A two-stage grounded emitter amplifier which produces approximately 90 db power gain is shown on the right and a micro-power audio oscillator is shown on the left.

a value corresponding to zero emitter voltage. A still further decrease in collector current can be obtained by inserting resistance between emitter and ground.

In order to increase the collector current to values higher than that corresponding to zero base current, a high resistance path between base and the positive supply voltage may be used as shown in Fig. 19. In this case the

collector current will increase by $1/(1 - \alpha)$ microamperes for each microampere which flows through the bias resistor. Since the current in the bias resistor will be approximately E_B/R, it is a simple matter to compute the required value of bias resistor once the desired collector current is known.

Figure 20 shows a two-stage audio amplifier which gives approximately 90 db gain. The circuit is shown in Fig. 21.

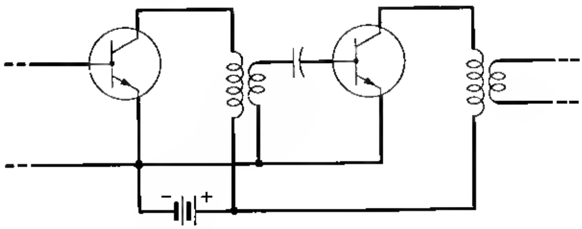

Fig. 21—Circuit of the amplifier shown in Fig. 20.

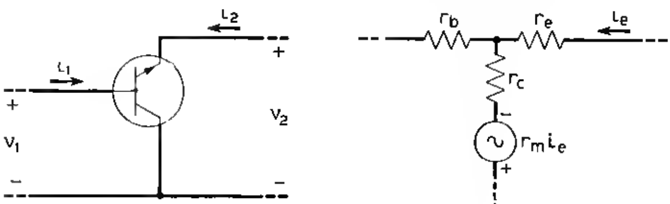

Fig. 22—The grounded collector connection of a transistor and the equivalent circuit.

The Grounded-Collector Stage

Although the power gain obtainable from this connection is relatively low—of the order of 15 or 20 db—it has very interesting possibilities in producing very high input impedances or very low output impedances. If it is worked into a fairly high load impedance, the input impedance may be several megohms, or if it is worked from a source of moderately low impedance (a few thousand ohms), the output impedance may be of the order of 25 ohms or lower.

For this type of stage, which is shown schematically in Fig. 22,

$$\begin{aligned} R_{11} &= r_b + r_c = 13.4\ (10)^6 \text{ ohms} \\ R_{12} &= r_c - r_m = 0.288\ (10)^6 \text{ ohms} \\ R_{21} &= r_c = 13.4\ (10)^6 \text{ ohms} \\ R_{22} &= r_e + r_c - r_m = 0.288\ (10)^6 \text{ ohms} \end{aligned} \tag{48}$$

If this stage is worked from a zero impedance generator into an infinite impedance load

$$v_2 = v_g\left(\frac{r_c}{r_b + r_c}\right) \tag{49}$$

$$\doteq v_g$$

and so, like a cathode follower, it gives an output voltage which is less than the input voltage, but in the same phase.

If the stage is operated into a short circuit

$$i_2 = -i_1 \frac{r_c}{r_e + r_c - r_m} \tag{50}$$

$$\doteq -i_1 \frac{1}{1 - \alpha} \tag{51}$$

$$= -46.5\ i_1$$

which indicates that the stage can give an appreciable current gain.

The input impedance is

$$R_i = r_b + r_c - \frac{r_c(r_c - r_m)}{r_e + r_c - r_m + R_L} \tag{52}$$

When $R_L = 0$, this reduces to

$$R_i = r_b + r_e \frac{1}{\dfrac{r_e}{r_c} + 1 - \dfrac{r_m}{r_c}} \tag{53}$$

$$\doteq r_b + r_e \frac{1}{1 - \alpha} \tag{54}$$

$$= 1445 \text{ ohms.}$$

When R_L is infinite

$$R_i = r_b + r_c \tag{55}$$

$$= 13.4(10)^6 \text{ ohms.}$$

With respect to input impedance, the grounded collector stage is again seen to be like a cathode follower in that the input impedance is high when tbe load impedance is high.

The output impedance is

$$R_o = r_e + r_c - r_m - \frac{r_c(r_c - r_m)}{r_b + r_c + R_g} \tag{56}$$

For $R_g = 0$, this reduces to

$$R_o = r_e + r_b \frac{r_c - r_m}{r_b + r_c} \tag{57}$$

$$\begin{aligned} &\doteq r_e + r_b(1 - \alpha) \\ &= 31.1 \text{ ohms.} \end{aligned} \tag{58}$$

For R_g infinite

$$\begin{aligned} R_o &= r_e + r_c - r_m \\ &= 0.288(10)^6 \text{ ohms.} \end{aligned} \tag{59}$$

The matched input impedance is

$$R_{im} = (r_b + r_c) \sqrt{\frac{r_b}{r_b + r_c} + \frac{r_e r_c}{(r_b + r_c)(r_e + r_c - r_m)}} \tag{60}$$

$$\begin{aligned} &\doteq \sqrt{r_c[r_b + r_e/(1 - \alpha)]} \\ &= 139{,}000 \text{ ohms.} \end{aligned} \tag{61}$$

The matched output impedance is

$$R_{om} = (r_e + r_c - r_m) \sqrt{\frac{r_b}{r_b + r_c} + \frac{r_e r_c}{(r_b + r_c)(r_e + r_c - r_m)}} \tag{62}$$

$$\begin{aligned} &\doteq (1 - \alpha) \sqrt{r_c[r_b + r_e/(1 - \alpha)]} \\ &= 2990 \text{ ohms.} \end{aligned} \tag{63}$$

With matched impedance, the maximum available gain of the grounded collector stage is

$$\text{M.A.G.} = \frac{\dfrac{r_c^2}{(r_b + r_c)(r_e + r_c - r_m)}}{\left[1 + \sqrt{\dfrac{r_b}{r_b + r_c} + \dfrac{r_e r_c}{(r_b + r_c)(r_e + r_c - r_m)}}\right]^2} \tag{64}$$

As α approaches unity, this approaches approximately

$$\text{M.A.G.} = r_c/4r_e \tag{65}$$

but so long as $r_e \ll r_c - r_m$, a good approximation is

$$\begin{aligned} \text{M.A.G.} &= 1/(1 - \alpha) \\ &= 46.5 \text{ or } 16.7 \text{ db.} \end{aligned} \tag{66}$$

The considerations involved in supplying biases to a grounded collector stage are rather similar to those discussed already for the grounded emitter case. If the base is allowed to float, the collector current will be given approximately by equation (47) as discussed for the grounded emitter case. A resistance between base and the negative side of the supply battery in Fig. 24 will serve to decrease the collector current while a resistance between base and ground will serve to increase it. In applications where it is desired to make full use of the high input impedance which this stage can afford, it may be most desirable to let the base float as shown in Fig. 23.

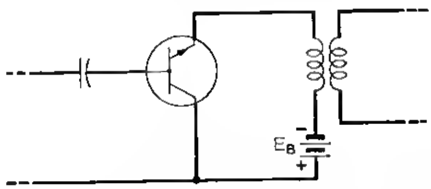

Fig. 23—One practical arrangement of a grounded collector stage.

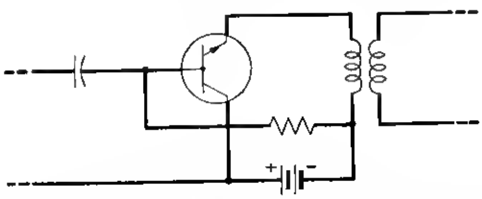

Fig. 24—Modification of Fig. 23 to obtain lower collector current. To raise collector current remove the resistance shown and connect a high resistance between base and ground.

Frequency Response—General Remarks

Shockley has shown that there are several different physical considerations which lead one to expect a high-frequency cutoff in the response of *n-p-n* transistors. The frequency at which cutoff occurs depends in a theoretically understandable way on such things as the geometry of the transistor and the physical properties of the germanium from which it is made. If these factors could all be controlled and varied at will, it would be possible to design a transistor to have a specified cutoff frequency.

One limitation comes about in the following way: In order to produce transistor action, the electrons which are injected into the *p* layer at the emitter junction must travel across this thin layer and arrive at the collector junction. They do this principally by a process of diffusion and require a finite (but small) amount of time to make the journey. If this time were

exactly the same for all electrons, the effect would be simply to delay the output signal with respect to the input and there would be no effect on frequency response. But there is a certain amount of dispersion in transit time which means that the electrons corresponding to a particlar part of the input signal wave do not all arrive simultaneously at the collector. When this difference in time of arrival amounts to an appreciable part of a cycle there is a tendency for some of the electrons to cancel the effect of others so that the frequency response begins to fall off. As the signal frequency increases beyond this point, the effect becomes more and more pronounced and the response continues to fall with increasing frequency.

In terms of the equivalent circuit, this dispersion in transit time means that beyond a certain frequency, r_m (and hence α) begins to decrease with increasing frequency and so the transistor may be said to have a certain α-cutoff which we will call $f_{c\alpha}$.

Shockley has shown that $f_{c\alpha}$ is inversely proportional to the square of the *p*-layer thickness and hence increases rapidly as the *p* layer is made thinner. For *n-p-n* transistors now available, this cutoff should occur at frequencies between five and twenty megacycles.

Another limitation on frequency response comes about from the fact that, at sufficiently high frequencies, the emitter junction fails to behave as a pure resistance and is, in effect, shunted by a capacitance. In terms of the equivalent circuit, this means that r_e is shunted by a capacitance.

The effect which this has on frequency response can be reduced by reducing the impedance of the source from which the emitter is driven. But so far as the emitter junction is concerned, r_b is always in series with the source impedance and so it is the value of r_b which ultimately determines the emitter cutoff frequency.

This capacitative reactance should begin to become appreciable with respect to emitter resistance at a frequency which may be of the same order as $f_{c\alpha}$. If r_b is high, the emitter cutoff frequency f_{ce} will then be of the same order of magnitude as $f_{c\alpha}$ and will increase as r_b is decreased.

A third cause for limited frequency response is the capacitance of the collector junction. The *n*-type germanium on one side of the junction behaves as one plate of a parallel-plate condenser and the *p*-type germanium on the other side behaves as the other plate. Since the transition from *n* to *p* type germanium may be made in an exceedingly small fraction of an inch, the plates of the condenser are very closely spaced and the capacitance may be appreciable.

Collector capacitance also depends on collector voltage, decreasing with increasing voltage. Theoretically, the capacitance should be in proportion to the negative one-third power of V_c.

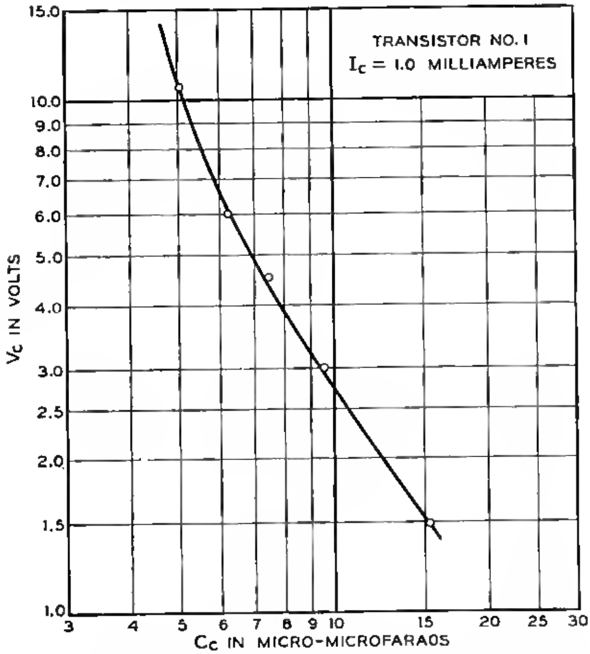

Fig. 25—Collector capacitance decreases as collector voltage is increased.

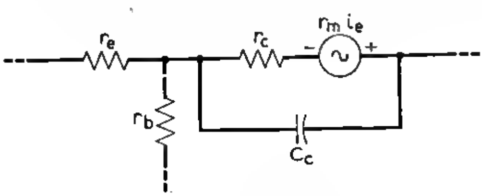

Fig. 26—The equivalent circuit of a transistor with collector capacitance shown.

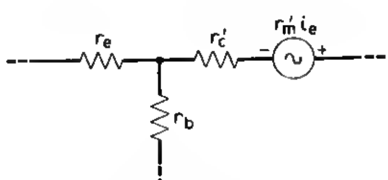

Fig. 27—The effect of collector capacitance is to change r_m and r_c to r_m' and r_c'. See equations (67) and (68).

Figure 25 shows measured values of C_c as a function of collector voltage. For reasons which are not understood at present, these data show a departure from the usual inverse one-third power variation. At V_c = 4.5 volts

the capacitance is seen to be approximately 7 micro-microfarads. In terms of the equivalent circuit, this capacitance is in shunt with the series combination of r_c and the generator, $r_m i_e$, as shown in Fig. 26. This can be shown to be equivalent to the circuit of Fig. 27 in which r_c has been replaced by

$$r_c' = r_c/(1 + jC_c r_c \omega) \tag{67}$$

and r_m has been replaced by

$$r_m' = r_m/(1 + jC_c r_c \omega). \tag{68}$$

The effect of collector capacitance can be computed by substituting r_c' and r_m' for the values of r_m and r_c (implicitly contained) in equation (8). In the sections which follow, this will be done for each of the three transistor connections and the resulting collector cutoff frequencies f_{cc} will be computed. It will be shown that at least for the transistor on which data are presented collector capacitance tends to produce a cutoff frequency well below those to be expected from emitter cutoff or alpha cutoff. For this reason, only collector cutoff will be considered.

Collector Cutoff in the Grounded Base Stage

If the values of r_c' and r_m' from equations (67) and (68) are substituted for r_c and r_m in equations (16) and the resulting values of the R's are substituted into equation (8), the result is

$$v_2/v_g = \frac{\alpha R_L}{(r_e + r_b + R_g)[1 + R_L(1 + j\omega C_c r_c)/r_c]} \tag{69}$$

The cutoff frequency f_{cc} is defined as the frequency at which the voltage across the load has dropped 3 db compared to its low-frequency value. This is the frequency at which the imaginary part of the denominator of (69) is equal to the real part. Solving for f_{cc} gives

$$f_{cc} = \frac{1}{2\pi C_c}\left[\frac{1}{R_L} + \frac{1}{r_c} - \frac{\alpha r_b}{R_L(r_e + r_b + R_g)}\right] \tag{70}$$

Substituting into this equation $C_c = 7(10)^{-12}$ farad, numerical values of the r's from (16), and the values $R_g = 91$ and $R_L = 4.58\ (10)^6$ ohms, corresponding to maximum available gain gives

$$f_{cc} = 3390 \text{ cps.}$$

With these terminations, the low-frequency gain is 44.3 db. If R_g and R_L are reduced to 25 and 200,000 ohms, respectively, f_{cc} is raised to 23,500 cps and the gain is lowered to 37.2 db. A further reduction of R_L to 20,000 ohms increases f_{cc} to 0.22 megacycles and reduces the gain to 27.8 db. This corre-

sponds to a gain-bandwidth product of $1.2(10)^8$ cps and shows that useful gain could be obtained at frequencies well above a megacycle, ***provided*** alpha and emitter cutoffs did not interfere.

Collector Cutoff in the Grounded-Emitter Stage

The procedure described in the last section leads, in this case, to

$$v_2/v_G = \frac{-R_L r_m/r_c + (R_L r_e/r_c)(1 + jr_c C_c \omega)}{r_e + (r_b + R_g)(1 - r_m/r_c) + [r_e R_L/r_c + (r_b + R_g)(r_e + R_L)/r_c](1 + jr_c C_c \omega)} \tag{71}$$

For the transistor of our numerical example, the imaginary term in the numerator is completely negligible at frequencies below $(10)^9$ cps. Neglecting it leads to

$$f_{cc} = \frac{1}{2\pi C_c} \frac{1 + R_L/r_c + [(r_b + R_g)/r_c][1 - (r_m - r_e - R_L)/r_c]}{R_L + [(r_b + R_g)/r_c](r_e + R_L)} \tag{72}$$

In this case the values of R_g and R_L (619 ohms and 671,000 ohms respectively) which correspond to maximum available gain give

$$f_{cc} = 3740 \text{ cps and}$$

$$\text{M.A.G.} = 53 \text{ db.}$$

Reducing R_L to 100,000 and increasing R_g to 1000 ohms gives

$$f_{cc} = 11{,}120 \text{ cps}$$

$$G = 50 \text{ db.}$$

For $R_g = 1000$ and $R_L = 10{,}000$,

$$f_{cc} = 97{,}900 \text{ cps}$$

$$G = 41.3 \text{ db.}$$

and for $R_g = R_L = 1000$ ohms,

$$f_{cc} = 943{,}000 \text{ cps}$$

$$G = 31.4 \text{ db.}$$

The gain-bandwidth product for this stage is $1.3(10)^9$ cps as compared to $1.2(10)^8$ cps for the same transistor connected as a grounded base amplifier. It should be pointed out, however, that this stage is particularly sensitive to change in α and on this account alpha cutoff may influence the response at fairly low frequencies. For example, when the terminating resistances are both 1000 ohms, reducing α from 0.9785 to 0.900 reduces the gain from 31.4 db to 0.2 db.

Collector Cutoff in the Grounded Collector Stage

In this case

$$v_2/v_g = \frac{R_L}{[r_e + R_L + (r_b + R_g)(1 - r_m/r_c)] + (1/r_c)(r_b + R_g)(r_e + R_L)(1 + j\omega C_c r_c)} \quad (73)$$

and

$$f_{cc} = \frac{1}{2\pi C_c}\left[\frac{1}{r_c} + \frac{1}{r_b + R_g} + \frac{1 - r_m/r_c}{r_e + R_L}\right] \quad (74)$$

For matched impedances ($R_g = 139{,}000$ ohms and $R_L = 2990$ ohms),

$$f_{cc} = 320{,}000 \text{ cps}$$

$$G = 16.7 \text{ db.}$$

The cutoff frequency can be raised by decreasing eitber R_g or R_L. With $R_g = 139{,}000$ and $R_L = 25$ ohms

$$f_{cc} = 9.77 \text{ megacycles}$$

$$G = 1.8 \text{ db}$$

The gain-bandwidth product in this case is $1.5(10)^7$.

Noise

The data now available on noise are insufficient to give an adequate picture of the performance of *n-p-n* transistors in this respect. Sucb measurements as have been made, however, make it clear that these devices are very much quieter than early point-contact transistors reported on by Ryder and Kircher.

Transistor noise seems still to decrease with increasing frequency at a rate of something like 11 db per decade. It also decreases as tbe thickness of the *p* layer is decreased.

Of the order of half a dozen units of various dimensions have been measured at 1000 cps and have shown noise figures as low as 8 db and as high as 25 db.

The dependence of noise figure on operating point has been measured for only one transistor. As indicated in Fig. 28 and Fig. 29, tbese data show that the noise figure improves as V_c is reduced and that it may be roughly independent of collector current. These data were taken on a grounded emitter stage with impedance match at the input terminals. Noise figure for this connection varies slightly with source impedance and has been found

to be a minimum when the source impedance is roughly equal to the input impedance of the stage.

It must be emphasized that this functional dependence of noise figure on operating point and source impedance has been measured for only one transistor. Further measurements may show that these results are not typical.

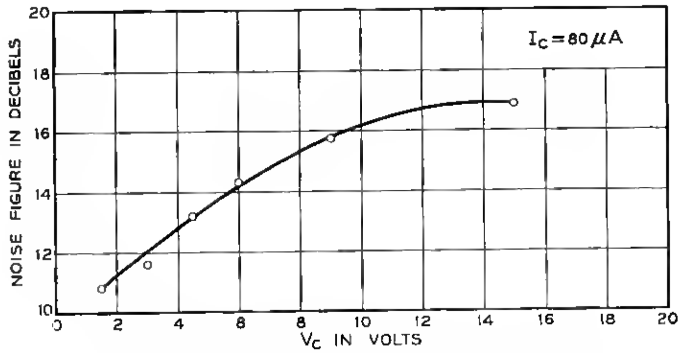

Fig. 28—Noise figure increases with increasing collector voltage.

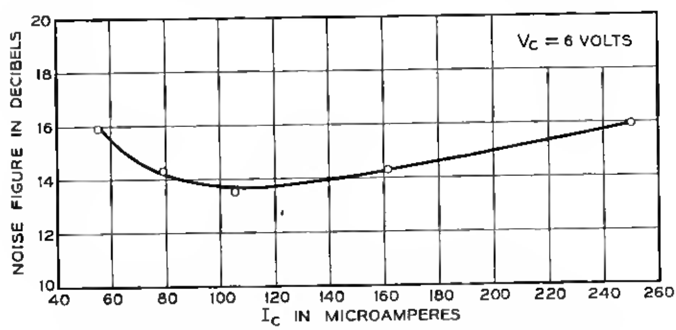

Fig. 29—Noise figure does not vary much with collector current.

Final Comments

In this paper we have attempted to present what is known about the circuit performance of *n-p-n* transistors. Since these devices are still undergoing exploratory development and since only a limited number has been produced, it is obviously impossible to give statistical data on reproducibility or on such reliability factors as the effect of ambient temperature.

It is much too soon to know what properties may be achieved after further development, but the results obtained to date seem encouraging and worth reporting.

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References

1. W. Shockley, "The Theory of *p-n* Junctions in Semiconductors and *p-n* Junction Transistors," *B. S. T. J.*, XXVIII, 435 (1949).
2. W. Shockley, "Electrons and Holes in Semiconductors," Van Nostrand (1950).
3. F. S. Goucher, G. L. Pearson, M. Sparks, G. K. Teal, W. Shockley, "Theory and Experiment for a Germanium *p-n* Junction," *Phys. Rev.*, *81*, 637 (1951).
4. W. Shockley, M. Sparks, G. K. Teal, "*p-n* Transistors," *Phys. Rev.*, *83*, 151 (1951).
5. R. M. Ryder, R. J. Kircher, "Some Circuit Aspects of the Transistor," *B. S. T. J.*, XXVIII, 367 (1949).